



**ΑΠΑΝΤΗΣΕΙΣ ΣΤΟ ΔΙΑΓΩΝΙΣΜΑ ΜΑΘΗΜΑΤΙΚΩΝ ΚΑΤΕΥΘΥΝΣΗΣ
Β' ΛΥΚΕΙΟΥ**

Κυριακή 22 Δεκεμβρίου 2013

ZHTHMA 1ο:

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|------------|---|
| 1. | Λ |
| 2. | Σ |
| 3. | Λ |
| 4. | Σ |
| 5. | Λ |
| 6. | Σ |
| 7. | Λ |
| 8. | Σ |
| 9. | Λ |
| 10. | Σ |

ZHTHMA 2ο:

A. i) β

$$\text{ii) } \bullet \text{ Μέσο της ΒΓ } \alpha \text{ Q } x_M = \frac{7+4}{2} = \frac{11}{2}, \quad y_M = \frac{3+5}{2} = 4. \text{ Άρα } M\left(\frac{11}{2}, 4\right)$$

$$\bullet \lambda_{AM} = \frac{4-1}{\frac{11}{2}-8} = \frac{3}{-\frac{5}{2}} = -\frac{6}{5}. \text{ Οπότε AM: } y-4 = -\frac{6}{5}\left(x-\frac{11}{2}\right) \Leftrightarrow y = -\frac{6}{5}x + \frac{53}{5}$$

B. Για $\lambda=0 \rightarrow 3x+y+1=0 \varepsilon 1$

$\lambda=1 \rightarrow 6x+y+4=0 \varepsilon 2$

$$\begin{cases} 3x+y+1=0 \\ 6x+y+4=0 \end{cases} \begin{cases} -3+y+1=0 \Leftrightarrow y=2 \\ 3x+3=0 \Leftrightarrow x=1 \end{cases} M(-1, 2)$$

$$(2\lambda^2 + \lambda + 3)(-1) + (\lambda^2 - \lambda + 1)2 + 3\lambda + 1 = 0$$

$$-2\lambda^2 - \lambda - 3 + 2\lambda^2 - 2\lambda + 2 + 3\lambda + 1 = 0$$

$$0\lambda^2 + 0\lambda + 0 = 0 \text{ ισχύει για κάθε } \lambda \in \mathbb{R}$$

ZHTHMA 3ο:

$$\alpha) \vec{\alpha} \cdot \vec{\beta} = \vec{\alpha} \cdot \pi \rho o \vec{\beta}_{\vec{\alpha}} = \vec{\alpha} \cdot \frac{5}{8} \cdot \vec{\alpha} = \frac{5}{8} \cdot \vec{\alpha}^2 = \frac{5}{8} \cdot |\vec{\alpha}|^2 = \frac{5}{8} \cdot 16 = 10$$

$$\beta) \sigma_{uv}(\vec{\alpha}, \vec{\beta}) = \frac{\vec{\alpha} \cdot \vec{\beta}}{|\vec{\alpha}| |\vec{\beta}|} = \frac{10}{4 \cdot 5} = \frac{1}{2} \text{ αQα } (\vec{\alpha}, \vec{\beta}) = 60^\circ$$

$$\gamma) |u|^2 = (\vec{\alpha} - \vec{\beta})^2 = \vec{\alpha}^2 - 2\vec{\alpha} \cdot \vec{\beta} + \vec{\beta}^2 = |\vec{\alpha}|^2 - 2 \cdot 10 + |\vec{\beta}|^2 = \\ = 16 - 20 + 25 = 21 \text{ αQα } |u| = \sqrt{21}$$

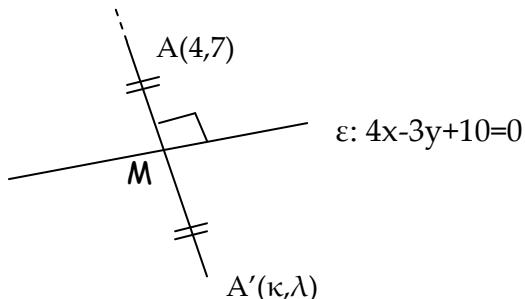
$$\delta) \vec{v} = 10 \cdot \vec{\alpha} - \kappa \vec{\beta}. \text{ Aφoύ } \vec{v} \perp \vec{\beta} \text{ τότε } \vec{v} \cdot \vec{\beta} = 0 \text{ δηλ. } (10 \vec{\alpha} - \kappa \vec{\beta}) \cdot \vec{\beta} = 0 \Leftrightarrow 10 \vec{\alpha} \cdot \vec{\beta} - \kappa \vec{\beta}^2 = 0 \\ \kappa |\vec{\beta}|^2 = 10 \cdot 10 \Leftrightarrow \kappa \cdot 25 = 100. \text{ AQα } \kappa = 4.$$

ZHTHMA 4ο:

A.

$$\text{i)} d(A, \varepsilon) = \frac{|4(\lambda - 2) + (-3)(\lambda + 1) + 10|}{\sqrt{4^2 + (-3)^2}} = 1 \quad \eta \\ \frac{|4\lambda - 8 - 3\lambda - 3 + 10|}{\sqrt{25}} = 1 \quad \eta \\ \begin{cases} \lambda - 1 = 5 \\ \lambda - 1 = -5 \end{cases} \quad \begin{cases} \lambda = 6 \\ \lambda = -4 \end{cases}$$

ii) Για $\lambda = 6$



$$\varepsilon_1 \perp \varepsilon \rightarrow \lambda \varepsilon_1 \cdot \lambda \varepsilon = -1$$

$$\lambda \varepsilon_1 \frac{4}{3} = -1 \text{ αQα } \lambda \varepsilon_1 = -\frac{3}{4}$$

$$\text{Αρχικά } \varepsilon 1: y - 7 = -\frac{3}{4}(x - 4)$$

$$4y - 28 = -3x + 12$$

$$\boxed{3x + 4y - 40 = 0}$$

$$\begin{cases} 3x + 4y - 40 = 0 \\ 4x - 3y + 10 = 0 \end{cases} \Leftrightarrow \begin{cases} 9x + 12y - 120 = 0 \\ 16x - 12y + 40 = 0 \end{cases}$$

$$25x - 80 = 0 \Leftrightarrow x = \frac{80}{25} = \frac{16}{5}$$

$$3 \cdot \frac{16}{5} + 4y - 40 = 0 \Leftrightarrow 4y = 40 - \frac{48}{5} \Leftrightarrow 4y = \frac{152}{5} \Leftrightarrow y = \frac{38}{5}$$

$$\text{Άρχικα } M \left(\frac{16}{5}, \frac{38}{5} \right)$$

$$\frac{4+\kappa}{2} = \frac{16}{5} \rightarrow 20 + 5\kappa = 32 \Leftrightarrow 5\kappa = 12 \Leftrightarrow \kappa = \frac{12}{5}$$

$$\frac{7+\lambda}{2} = \frac{38}{5} \rightarrow 35 + 5\lambda = 76 \Leftrightarrow 5\lambda = 41 \Leftrightarrow \lambda = \frac{41}{5}$$

$$A' \left(\frac{12}{5}, \frac{41}{5} \right)$$

$$\text{iii) } \begin{cases} x = \lambda - 2 \\ y = \lambda + 1 \end{cases} \quad \begin{cases} x + 2 = \lambda \\ y - 1 = \end{cases}$$

$$\text{Άρχικα } x + 2 = y - 1 \Leftrightarrow$$

$$x - y + 3 = 0$$

Οπότε ο Γεωμετρικός τόπος των σημείων A είναι η ευθεία $x - y + 3 = 0$.