α) Η διακρίνουσα του τριωνύμου $\alpha x^2 - (\alpha^2 - 1)x - \alpha$ είναι:

$$\Delta = \beta^2 - 4\alpha\gamma = (-(\alpha^2 - 1))^2 - 4 \cdot \alpha \cdot (-\alpha) =$$
$$= \alpha^4 - 2\alpha^2 + 1 + 4\alpha^2 =$$
$$= \alpha^4 + 2\alpha^2 + 1 = (\alpha^2 + 1)^2$$

β) Είναι $\Delta > 0$ οπότε η εξίσωση έχει δύο άνισες ρίζες:

$$x_{1,2} = \frac{-\beta \pm \sqrt{\Delta}}{2\alpha} = \frac{-\left(-(\alpha^2-1)\right) \pm \sqrt{(\alpha^2+1)^2}}{2 \cdot \alpha} = \frac{\alpha^2 - 1 \pm (\alpha^2+1)}{2\alpha} = \begin{cases} \frac{\alpha^2-1+\alpha^2+1}{2\alpha} = \alpha \\ \frac{\alpha^2-1-\alpha^2-1}{2\alpha} = -\frac{1}{\alpha} \end{cases}$$

γ) Είναι:

$$|\rho_1 - \rho_2| = 2 \Leftrightarrow \left|\alpha - \left(-\frac{1}{\alpha}\right)\right| = 2 \Leftrightarrow$$
$$\Leftrightarrow \left(\alpha + \frac{1}{\alpha} = -2 \text{ ή } \alpha + \frac{1}{\alpha} = 2\right) \Leftrightarrow$$
$$\Leftrightarrow (\alpha^2 + 1 = -2\alpha \text{ ή } \alpha^2 + 1 = 2\alpha) \Leftrightarrow$$
$$\Leftrightarrow (\alpha^2 + 2\alpha + 1 = 0 \text{ ή } \alpha^2 - 2\alpha + 1 = 0) \Leftrightarrow$$
$$\Leftrightarrow ((\alpha + 1)^2 = 0 \text{ ή } (\alpha - 1)^2 = 0) \Leftrightarrow$$
$$\Leftrightarrow (\alpha + 1 = 0 \text{ ή } \alpha - 1 = 0) \Leftrightarrow$$
$$\Leftrightarrow (\alpha = -1 \text{ ή } \alpha = 1)$$