α) Ισχύει ότι:

$$|s_\Delta - s_A| = |s_\Delta - s_B| \Leftrightarrow$$

$$\Leftrightarrow (s_\Delta - s_A = s_\Delta - s_B \ \ ή \ \ s_\Delta - s_A = -(s_\Delta - s_B)) \Leftrightarrow$$

$$\Leftrightarrow (s_A = s_B, \text{ απορρίπτεται} \ \ ή \ \ s_\Delta - s_A = -s_\Delta + s_B) \Leftrightarrow$$

$$\Leftrightarrow 2s_\Delta = s_A + s_B \Leftrightarrow s_\Delta = \frac{s_A + s_B}{2}$$

Είναι:

$$s_A - s_\Delta = s_A - \frac{s_A + s_B}{2} =$$

$$= \frac{2s_A - s_A - s_B}{2} = \frac{s_A - s_B}{2} < 0 \Leftrightarrow$$

$$\Leftrightarrow s_A < s_\Delta \ \ (1)$$

Ισχύει επίσης ότι:
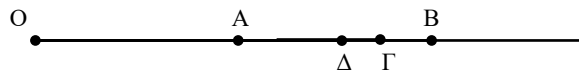
$$s_\Delta - s_\Gamma = \frac{s_A + s_B}{2} - \frac{s_A + 3s_B}{4} =$$

$$= \frac{2s_A + 2s_B - s_A - 3s_B}{4} = \frac{s_A - s_B}{4} < 0 \Leftrightarrow$$

$$\Leftrightarrow s_\Delta < s_\Gamma \ \ (2)$$

Είναι:

$$s_\Gamma - s_B = \frac{s_A + 3s_B}{4} - s_B =$$

$$= \frac{s_A + 3s_B - 4s_B}{4} = \frac{s_A - s_B}{4} < 0 \Leftrightarrow$$

$$\Leftrightarrow s_\Gamma < s_B \ \ (3)$$

Από τις σχέσεις (1), (2) και (3) βρίσκουμε:

$$s_A < s_\Delta < s_\Gamma < s_B$$

β) i) Μια εξίσωση $2^{ου}$ βαθμού είναι η:

$$x^2 - Sx + P = 0 \Leftrightarrow x^2 - (s_A + s_B)x + s_A s_B = 0 \Leftrightarrow$$

$$\Leftrightarrow x^2 - 1{,}4x + 0{,}45 = 0 \ (4)$$

ii) Η εξίσωση (4) έχει διακρίνουσα:

$$\Delta = \beta^2 - 4\alpha\gamma = (-1{,}4)^2 - 4 \cdot 1 \cdot 0{,}45 = 0{,}16$$

και ρίζες τις

$$x_{1,2} = \frac{-\beta \pm \sqrt{\Delta}}{2\alpha} = \frac{-(-1{,}4) \pm \sqrt{0{,}16}}{2 \cdot 1} = \frac{1{,}4 \pm 0{,}4}{2} = \begin{cases} \frac{1{,}4 + 0{,}4}{2} = 0{,}9 \\ \frac{1{,}4 - 0{,}4}{2} = 0{,}5 \end{cases}$$

Επειδή ισχύει $s_A < s_B$ είναι:

$$s_A = 0{,}5 \text{ km} \ \ και \ \ s_B = 0{,}9 \text{ km}$$

Τέλος βρίσκουμε:

$$s_\Gamma = \frac{s_A + 3s_B}{4} = \frac{0{,}5 + 3 \cdot 0{,}9}{4} = 0{,}8 \text{ km}$$

$$s_\Delta = \frac{s_A + s_B}{2} = \frac{0{,}5 + 0{,}9}{2} = 0{,}7 \text{ km}$$

$$s_\Gamma = \frac{s_A + 3s_B}{4} = \frac{0{,}5 + 3 \cdot 0{,}9}{4} = 0{,}8 \text{ km}$$

$$s_\Delta = \frac{s_A + s_B}{2} = \frac{0{,}5 + 0{,}9}{2} = 0{,}7 \text{ km}$$