α) Είναι:

$$\alpha_3 = 8 \Leftrightarrow \alpha_1 + (3-1)\omega = 8 \Leftrightarrow$$

$$\Leftrightarrow \alpha_1 + 2\omega = 8 \Leftrightarrow \alpha_1 = 8 - 2\omega \quad (1)$$

Επίσης ισχύει ότι:

$$\alpha_8 = 23 \Leftrightarrow \alpha_1 + (8-1)\omega = 23 \Leftrightarrow$$

$$\Leftrightarrow \alpha_1 + 7\omega = 23 \overset{(1)}{\Leftrightarrow}$$

$$\Leftrightarrow 8 - 2\omega + 7\omega = 23 \Leftrightarrow$$

$$\Leftrightarrow 5\omega = 15 \Leftrightarrow \omega = 3$$

Αντικαθιστούμε στη σχέση (1) και βρίσκουμε:

$$\alpha_1 = 8 - 2 \cdot 3 \Leftrightarrow \alpha_1 = 2$$

β) Είναι:

$$\alpha_{31} = \alpha_1 + (31-1)\omega =$$

$$=2 + 30 \cdot 3 = 2 + 90 = 92$$

γ) Έχουμε:

$$S = (\alpha_1 + 1) + (\alpha_2 + 2) + (\alpha_3 + 3) + ...+ (\alpha_{31} + 31) =$$

$$= (\alpha_1 + \alpha_2 + \alpha_3 + \ldots + \alpha_{31}) + (1 + 2 + 3 + \ldots + 31) = S_1 + S_2$$

όπου

$S_1 = \alpha_1 + \alpha_2 + \alpha_3 + \ldots + \alpha_{31} = \frac{31}{2}[2 \cdot 2 + (31-1) \cdot 3] = \frac{31}{2}(4 + 30 \cdot 3) = \frac{31}{2} \cdot 94 = 1457$

$S_2 = 1 + 2 + 3 + \ldots + 31 = \frac{31}{2}(1 + 31) = \frac{31}{2} \cdot 32 = 496$

Τελικά:

$$S = 1497 + 496 = 1953$$