

α) Είναι:

$$\begin{aligned}\alpha_3 = 8 &\Leftrightarrow \alpha_1 + (3 - 1)\omega = 8 \Leftrightarrow \\ \Leftrightarrow \alpha_1 + 2\omega = 8 &\Leftrightarrow \alpha_1 = 8 - 2\omega \quad (1)\end{aligned}$$

Επίσης ισχύει ότι:

$$\begin{aligned}\alpha_8 = 23 &\Leftrightarrow \alpha_1 + (8 - 1)\omega = 23 \Leftrightarrow \\ &\Leftrightarrow \alpha_1 + 7\omega = 23 \stackrel{(1)}{\Leftrightarrow} \\ &\Leftrightarrow 8 - 2\omega + 7\omega = 23 \Leftrightarrow \\ &\Leftrightarrow 5\omega = 15 \Leftrightarrow \omega = 3\end{aligned}$$

Αντικαθιστούμε στη σχέση (1) και βρίσκουμε:

$$\alpha_1 = 8 - 2 \cdot 3 \Leftrightarrow \alpha_1 = 2$$

β) Είναι:

$$\begin{aligned}\alpha_{31} &= \alpha_1 + (31 - 1)\omega = \\ &= 2 + 30 \cdot 3 = 2 + 90 = 92\end{aligned}$$

γ) Έχουμε:

$$\begin{aligned}S &= (\alpha_1 + 1) + (\alpha_2 + 2) + (\alpha_3 + 3) + \dots + (\alpha_{31} + 31) = \\ &= (\alpha_1 + \alpha_2 + \alpha_3 + \dots + \alpha_{31}) + (1 + 2 + 3 + \dots + 31) = S_1 + S_2\end{aligned}$$

όπου

$$S_1 = \alpha_1 + \alpha_2 + \alpha_3 + \dots + \alpha_{31} = \frac{31}{2}[2 \cdot 2 + (31 - 1) \cdot 3] = \frac{31}{2}(4 + 30 \cdot 3) = \frac{31}{2} \cdot 94 = 1457$$

$$S_2 = 1 + 2 + 3 + \dots + 31 = \frac{31}{2}(1 + 31) = \frac{31}{2} \cdot 32 = 496$$

Τελικά:

$$S = 1457 + 496 = 1953$$