

α) • Είναι:

$$\begin{aligned} A + B &= \frac{1}{3-\sqrt{7}} + \frac{1}{3+\sqrt{7}} = \\ &= \frac{3+\sqrt{7}}{(3+\sqrt{7})(3-\sqrt{7})} + \frac{3-\sqrt{7}}{(3+\sqrt{7})(3-\sqrt{7})} = \\ &= \frac{3-\sqrt{7}+3+\sqrt{7}}{(3+\sqrt{7})(3-\sqrt{7})} = \\ &= \frac{6}{3^2-(\sqrt{7})^2} = \frac{6}{9-7} = \frac{6}{2} = 3 \end{aligned}$$

• Ισχύει ότι:

$$\begin{aligned} A \cdot B &= \frac{1}{3-\sqrt{7}} \cdot \frac{1}{3+\sqrt{7}} = \\ &= \frac{1}{(3+\sqrt{7})(3-\sqrt{7})} = \frac{1}{3^2-(\sqrt{7})^2} = \\ &= \frac{1}{9-7} = \frac{1}{2} \end{aligned}$$

β) Η ζητούμενη εξίσωση είναι της μορφής:

$$x^2 - Sx + P = 0$$

με

$$S = A + B = 3 \quad \text{και} \quad P = A \cdot B = \frac{1}{2}$$

Τελικά η ζητούμενη εξίσωση είναι η:

$$x^2 - 3x + \frac{1}{2} = 0 \Leftrightarrow 2x^2 - 6x + 1 = 0$$