

α) Η δοθείσα ισότητα ισοδύναμα γράφεται:

$$\begin{aligned}\frac{\alpha^2+1}{\beta^2+1} &= \frac{\alpha}{\beta} \Leftrightarrow \beta(\alpha^2+1) = \alpha(\beta^2+1) \Leftrightarrow \\ &\Leftrightarrow \alpha^2\beta + \beta = \alpha\beta^2 + \alpha \Leftrightarrow \\ &\Leftrightarrow \alpha^2\beta + \beta - \alpha\beta^2 - \alpha = 0 \Leftrightarrow \\ &\Leftrightarrow \alpha\beta(\alpha - \beta) - (\alpha - \beta) = 0 \Leftrightarrow \\ &\Leftrightarrow (\alpha - \beta)(\alpha\beta - 1) = 0 \stackrel{\alpha \neq \beta}{\Leftrightarrow} \\ &\Leftrightarrow \alpha\beta - 1 = 0 \Leftrightarrow \alpha\beta = 1\end{aligned}$$

Άρα οι αριθμοί α, β είναι αντίστροφοι.

β) Η παράσταση γράφεται:

$$K = \frac{\alpha^{22} \cdot (\beta^3)^8}{\alpha^{-2} \cdot (\alpha\beta)^{25}} = \frac{\alpha^{22} \cdot \beta^{24}}{\alpha^{-2} \cdot \alpha^{25} \cdot \beta^{25}} = \frac{\alpha^{22} \cdot \beta^{24}}{\alpha^{23} \cdot \beta^{25}} = \frac{\alpha^{22}}{\alpha^{23}} \cdot \frac{\beta^{24}}{\beta^{25}} = \frac{1}{\alpha} \cdot \frac{1}{\beta} = \frac{1}{\alpha\beta} = 1$$