α) Είναι:

- $f\left(\frac{1}{2}\right) = \frac{1}{2} + \frac{1}{\frac{1}{2}} = \frac{1}{2} + 2 = \frac{1}{2} + \frac{4}{2} = \frac{5}{2}$
- $f(1) = 1 + \frac{1}{1} = 1 + 1 = 2$
- $f(2) = 1 + \frac{1}{2} = \frac{2}{2} + \frac{1}{2} = \frac{3}{2}$

Τότε:

$$A = f\left(\frac{1}{2}\right) + f(1) - f(2) =$$
$$= \frac{5}{2} + 2 - \frac{3}{2} =$$
$$= \frac{2}{2} + 2 = 1 + 2 = 3$$

β) Ισχύει ότι:

$$f(x) = \frac{5}{2} \Leftrightarrow x + \frac{1}{x} = \frac{5}{2} \Leftrightarrow$$
$$\Leftrightarrow \frac{x^2+1}{x} = \frac{5}{2} \Leftrightarrow 2(x^2 + 1) = 5x \Leftrightarrow$$
$$\Leftrightarrow 2x^2 + 2 = 5x \Leftrightarrow 2x^2 - 5x + 2 = 0$$

Το τριώνυμο $2x^2 - 5x + 2$ έχει $\alpha = 2$, $\beta = -5$, $\gamma = 2$ και διακρίνουσα:

$$\Delta = \beta^2 - 4\alpha\gamma = (-5)^2 - 4 \cdot 2 \cdot 2 = 25 - 16 = 9 > 0$$

Οι ρίζες του τριωνύμου είναι οι:

$$x_{1,2} = \frac{-\beta \pm \sqrt{\Delta}}{2\alpha} = \frac{-(-5) \pm \sqrt{9}}{2 \cdot 2} = \frac{5 \pm 3}{4} = \begin{cases} \frac{5+3}{4} = 2 \\ \frac{5-3}{4} = \frac{1}{2} \end{cases}$$