

α) i) Είναι:

$$\begin{aligned} A + B &= \frac{1}{5+\sqrt{5}} + \frac{1}{5-\sqrt{5}} = = \\ &= \frac{5-\sqrt{5}}{(5+\sqrt{5})(5-\sqrt{5})} + \frac{5+\sqrt{5}}{(5+\sqrt{5})(5-\sqrt{5})} = \\ &= \frac{5-\sqrt{5}+5+\sqrt{5}}{(5+\sqrt{5})(5-\sqrt{5})} = \\ &= \frac{10}{5^2-(\sqrt{5})^2} = \frac{10}{25-5} = \\ &= \frac{10}{20} = \frac{1}{2} \end{aligned}$$

ii) Ισχύει ότι:

$$\begin{aligned} A \cdot B &= \frac{1}{5+\sqrt{5}} \cdot \frac{1}{5-\sqrt{5}} = \\ &= \frac{1}{(5+\sqrt{5})(5-\sqrt{5})} = \frac{1}{5^2-(\sqrt{5})^2} = \\ &= \frac{1}{25-5} = \frac{1}{20} \end{aligned}$$

β) Η ζητούμενη εξίσωση είναι της μορφής:

$$x^2 - Sx + P = 0$$

με

$$S = A + B = \frac{1}{2} \text{ και } P = A \cdot B = \frac{1}{20}$$

Τελικά η ζητούμενη εξίσωση είναι η:

$$x^2 - \frac{1}{2}x + \frac{1}{20} = 0 \Leftrightarrow 20x^2 - 10x + 1 = 0$$